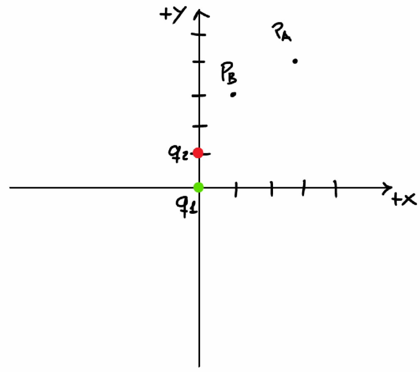
12. Supongamos dos partículas cargadas $q_1 = 10\ nC$ y $q_2 = -0.02\ \mu C$ colocadas en $P_1 = (0,0)\ m$ y $P_2 = (0,1)\ m$ respectivamente. Calcular el campo eléctrico producido por este sistema en los siguientes puntos del espacio:
a. $P_A = (3,4)\ m.$
b. $P_B = (1,3)\ m.$
c. $P_C = (2,0)\ m.$
d. $P_D = (0.5, 0.5)\ m.$



$q_1 = 10^{-8} C$

$q_2 = -2\cdot 10^{-8} C$

$P_1 = (0,0)$

$P_2 = (0,1)$

- Para calcular el campo eléctrico en los diferentes puntos, aplicamos la ley de Coulomb para el campo eléctrico debido a un sistema de partículas cargadas.
- Vamos a resolver cada apartado de un modo diferente, aunque todos ellos son equivalentes.

a) $P_A = (3,4)$

$$\bar{E}_{SP} = \sum_i \bar{E}_{iP} = \sum_i k \frac{q_i}{r_{iP}^2}\hat{r}_{iP}$$

$$\bar{E}_{1P} = \begin{cases} k\frac{q_1}{r_{1P}^2}\hat{r}_{1P} = k\frac{q_1}{r_{1P}^2}\left(\cos(\theta_1)\hat{\imath} + \mathrm{sen}(\theta_1)\hat{\jmath}\right) \\ E_{1P_x}\hat{\imath} + E_{1P_y}\hat{\jmath} = k\frac{q_1}{r_{1P}^2}\left(\cos(\theta_1)\hat{\imath} + \mathrm{sen}(\theta_1)\hat{\jmath}\right) \end{cases}$$

(Cualquiera de estas dos formas de expresar el campo eléctrico por componentes es válida.)

$$\bar{E}_{2P} = \begin{cases} k\frac{q_2}{r_{2P}^2}\hat{r}_{2P} = k\frac{q_2}{r_{2P}^2}\left(\cos(\theta_2)\hat{\imath} + \mathrm{sen}(\theta_2)\hat{\jmath}\right) \\ E_{2P_x}\hat{\imath} + E_{2P_y}\hat{\jmath} = k\frac{q_2}{r_{2P}^2}\left(\cos(\theta_2)\hat{\imath} + \mathrm{sen}(\theta_2)\hat{\jmath}\right) \end{cases}$$

$$\bar{E}_{SP} = k\frac{q_1}{r_{1P}^2}\left(\cos(\theta_1)\hat{\imath} + \mathrm{sen}(\theta_1)\hat{\jmath}\right) + k\frac{q_2}{r_{2P}^2}\left(\cos(\theta_2)\hat{\imath} + \mathrm{sen}(\theta_2)\hat{\jmath}\right)$$

$r_{1P} = \sqrt{3^2+4^2} = 5$ $\qquad$ $r_{2P} = \sqrt{3^2+3^2} = 3\sqrt{2}$

$\cos(\theta_1) = \frac{3}{5}$ $\qquad$ $\cos(\theta_2) = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\mathrm{sen}(\theta_1) = \frac{4}{5}$ $\qquad$ $\sin(\theta_2) = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}}$

$$\bar{E}_{SP} = k\frac{q_1}{25}\left(\frac{3}{5}\hat{\imath} + \frac{4}{5}\hat{\jmath}\right) + k\frac{q_2}{18}\left(\frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath}\right)$$

$$\boxed{\bar{E}_{SP} = -4.9\,\hat{\imath} - 4.2\,\hat{\jmath}\ N/C}$$

b) $P_B = (1,3)$

$$\bar{E}_{SP} = \sum_i \bar{E}_{iP} = \sum_i k \frac{q_i}{r_{iP}^2}\hat{r}_{iP}$$

$$\sum_i \bar{E}_{iP} = \bar{E}_{1P} + \bar{E}_{2P}$$

$$\bar{E}_{SP} = \bar{E}_{1P} + \bar{E}_{2P}$$

Expresamos la ecuación por componentes:

$$E_{SP_x} = E_{1P_x} + E_{2P_x}$$

$$E_{SP_y} = E_{1P_y} + E_{2P_y}$$

Calculamos el valor de cada componente:

$$E_{1P_x} = E_{1P}\cos(\theta_1) = k\frac{q_1}{r_{1P}^2}\cos(\theta_1) = k\frac{q_1}{1^2+3^2}\,\frac{1}{\sqrt{10}} = k\frac{q_1}{10\sqrt{10}}$$

$$E_{1P_y} = E_{1P}\,\mathrm{sen}(\theta_1) = k\frac{q_1}{r_{1P}^2}\mathrm{sen}(\theta_1) = k\frac{q_1}{1^2+3^2}\,\frac{3}{\sqrt{10}} = k\frac{3q_1}{10\sqrt{10}}$$

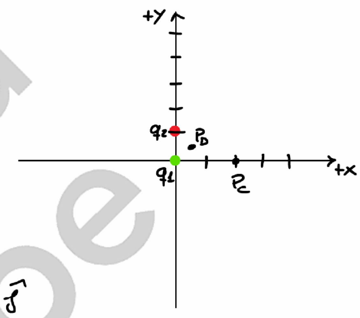
$$E_{2P_x} = E_2\cos(\theta_2) = k\frac{q_2}{r_{2P}^2}\cos(\theta_2) = k\frac{q_2}{1^2+2^2}\frac{1}{\sqrt{5}} = k\frac{q_2}{5\sqrt{5}}$$

$$E_{2P_y} = E_2\,\text{sen}(\theta_2) = k\frac{q_2}{r_{2P}^2}\sin(\theta_2) = k\frac{q_2}{1^2+2^2}\frac{2}{\sqrt{5}} = k\frac{2q_2}{5\sqrt{5}}$$

Por tanto:

$$\left.\begin{aligned} E_{SP_x} &= k\frac{q_1}{10\sqrt{10}} + k\frac{q_2}{5\sqrt{5}} = -13.2\ \text{N/C} \\ E_{SP_y} &= k\frac{3q_1}{10\sqrt{10}} + k\frac{2q_2}{5\sqrt{5}} = -23.6\ \text{N/C}. \end{aligned}\right\} \quad \boxed{\bar{E}_{SP} = -13.2\,\hat{\imath} - 23.6\,\hat{\jmath}\ \ \text{N/C}}$$

c) $P_c = (2,0)$

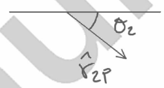


$$\bar{E}_{SP} = \sum_i \bar{E}_{iP} = \sum_i k\frac{q_i}{r_{iP}^2}\hat{r}_{iP}$$

$$\bar{E}_{1P}\begin{cases} E_{1P_x} = k\dfrac{|q_1|}{r_{1P}^2} = k\dfrac{|q_1|}{4} \\ E_{1P_y} = 0 \end{cases} \Rightarrow \bar{E}_{1P} = k\frac{|q_1|}{4}\hat{\imath}$$

$$\bar{E}_{2P} = \begin{cases} E_{2P_x} = k\dfrac{|q_2|}{r_{2P}^2}\cos(\theta_2) = k\dfrac{|q_2|}{5}\dfrac{2}{\sqrt{5}} \\ E_{2P_y} = k\dfrac{|q_2|}{r_{2P}^2}\text{sen}(\theta_2) = k\dfrac{|q_2|}{5}\dfrac{2}{\sqrt{5}} \end{cases} \Rightarrow \bar{E}_{2P} = -k\frac{2|q_2|}{5\sqrt{5}}\hat{\imath} + k\frac{2|q_2|}{5\sqrt{5}}\hat{\jmath}$$

(El sentido del campo lo introducimos mediante el signo que acompaña al módulo de cada componente.)



$$\bar{E}_{SP} = k\frac{|q_1|}{4}\hat{\imath} - k\frac{2|q_2|}{5\sqrt{5}}\hat{\imath} + k\frac{2|q_2|}{5\sqrt{5}}\hat{\jmath}$$

$$\boxed{\bar{E}_{SP} = -9.7\,\hat{\imath} + 16.1\,\hat{\jmath}\ \ \text{N/C}}$$

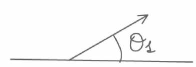
d) $P_D = (0.5, 0.5)$

$$\bar{E}_{SP} = \sum_i \bar{E}_{iP} = \sum_i k\frac{q_i}{r_{iP}^2}\hat{r}_{iP}$$

$$\bar{E}_{1P} = k\frac{q_1}{r_{1P}^2}\hat{r}_{1P};$$

$$r_{1P} = \sqrt{0.5^2+0.5^2} = \sqrt{0.5}$$

$$\hat{r}_{1P} = \cos(\theta_1)\hat{\imath} + \text{sen}(\theta_1)\hat{\jmath} = \frac{0.5}{\sqrt{0.5}}\hat{\imath} + \frac{0.5}{\sqrt{0.5}}\hat{\jmath} = \frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath}$$



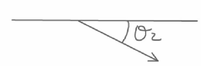
$$\bar{E}_{1P} = k2q_1\left(\frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath}\right) = k\frac{2}{\sqrt{2}}q_1(\hat{\imath}+\hat{\jmath}) = k\sqrt{2}q_1(\hat{\imath}+\hat{\jmath})$$

$$\bar{E}_{2P} = k\frac{q_2}{r_{2P}^2}\hat{r}_{2P};$$

$$r_{2P} = \sqrt{0.5^2+0.5^2} = \sqrt{0.5}$$

El signo es negativo porque es el seno de un ángulo negativo que se encuentra en el cuarto cuadrante.

$$\hat{r}_{2P} = \cos(\theta_2)\hat{\imath} + \text{sen}(\theta_2)\hat{\jmath} = \frac{0.5}{\sqrt{0.5}}\hat{\imath} - \frac{0.5}{\sqrt{0.5}}\hat{\jmath} = \frac{1}{\sqrt{2}}\hat{\imath} - \frac{1}{\sqrt{2}}\hat{\jmath}$$



$$\bar{E}_{2P} = k2q_2\left(\frac{1}{\sqrt{2}}\hat{\imath} - \frac{1}{\sqrt{2}}\hat{\jmath}\right) = k\frac{2}{\sqrt{2}}q_2(\hat{\imath}-\hat{\jmath}) = k\sqrt{2}q_2(\hat{\imath}-\hat{\jmath})$$

$$\bar{E}_{SP} = k\sqrt{2}q_1(\hat{\imath}+\hat{\jmath}) + k\sqrt{2}q_2(\hat{\imath}-\hat{\jmath}) = k\sqrt{2}\left[(q_1+q_2)\hat{\imath} + (q_1-q_2)\hat{\jmath}\right]$$

$$\boxed{\bar{E}_{SP} = -127\,\hat{\imath} + 381\,\hat{\jmath}\ \ \text{N/C}}$$